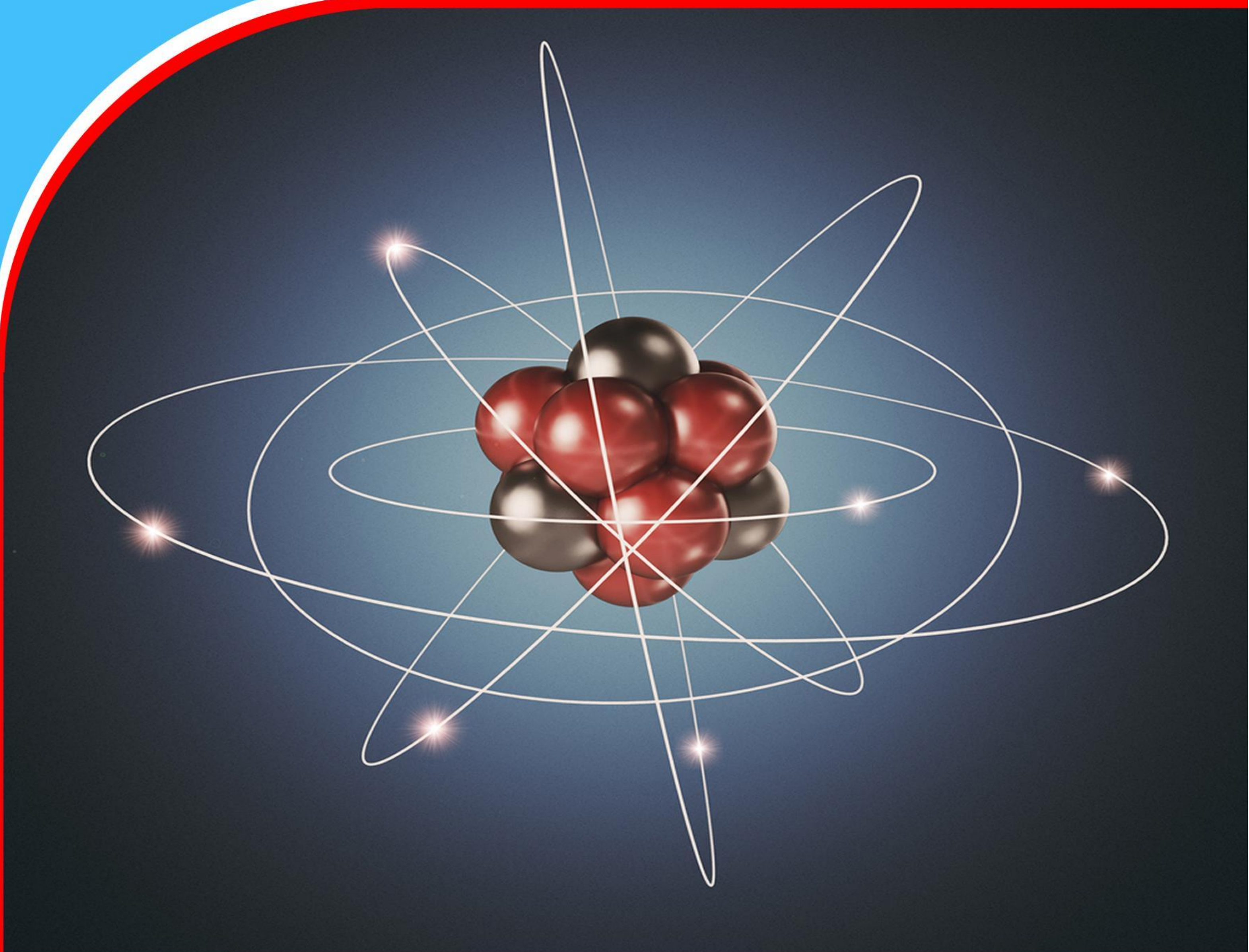
European Journal of
Physical Sciences
(EJPS)



APPLYING DUDUCTIONS FROM NAVIER STOKES EQUATION TO FLOW SITUATIONS IN GAS PIPELINE NETWORK SYSTEM

Mathew Shadrack Uzoma

European Journal of Physical Sciences
Vol.1, Issue 1 No.2, pp 10 - 18, 2019

APPLYING DUDUCTIONS FROM NAVIER STOKES EQUATION TO FLOW SITUATIONS IN GAS PIPELINE NETWORK SYSTEM

Mathew Shadrack Uzoma
Department of Mechanical Engineering
University of Port Harcourt
Port Harcourt, Rivers State, Nigeria
E-mail: matshadrack@yahoo.com

Abstract

Navier Stokes equations are theoretical equations for pressure-flow-temperature problems in gas pipelines. Other well-known gas equations such as Weymouth, Panhandle A and Modified Panhandle B equations are employed in gas pipeline design and operational procedures at a level of practical relevance. Attaining optimality in the performance of this system entails concrete understanding of the theoretical and prevailing practical flow conditions. In this regard, Navier Stoke's mass, momentum and energy equations had been worked upon subject to certain simplifying assumptions to deduced expressions for flow velocity and throughput in gas pipeline network system. This work could also bridge the link among theoretical, operational and optimal level of performance in gas pipelines.

Purpose: The purpose of this research is to build a measure of practical relevance in gas pipeline operational procedures that would ultimately couple the missing links between theoretical flow equations such as Navier Stokes equation and practical gas pipeline flow equations. Such practical gas pipeline flow models are Weymouth, Panhandle A and Modified Panhandle B equations among others.

Methodology: The approach in this regard entails reducing Narvier Stoke's mas, momentum and energy equations to their appropriate forms by applicable practical conditions. By so doing flow models are deduced that could be worked upon by computational approach analytically or numerically to determine line throughput and flow velocity.The reduced forms of the Navier Stokes velocity and throughput equations would be applied to operating gas pipelines in Nigeria terrain. The gas pipelines are ElfTotal Nig. Ltd and Shell Petroleum Development Company (SPDC). This would enable the comparison of these gas pipelines operational data with theoretical results of Navier Stokes equations reduced to their appropriate forms.

Findings: The follow up paper would employ theoretical and numerical discretization computational methods to compare theoretical and numerical discretization results to give a clue if these operating gas pipelines are operated at optimal level of performance.

Unique contribution to theory, practice and policy: The reduced forms of Nervier Stokes equations applied to physical operating gas pipelines network system is considered by the researcher to be an endeavor of academic excellence that would foster clear cut understanding of theoretical and practical flow situations. It could also open up a measure of understanding to pushing a flow to attaining optical conditions in practical real life flow situations. Operating gas pipelines optimally would reduce the spread of these capital intensive assets and facilities and more so conserving our limited reserves for foreign exchange.

Keywords: *Navier Stokes equations; Pressure-flow-temperature problems; Weymouth, Panhandle A and Modified Panhandle equations; Practical relevance; Flow velocity and throughput.*

NOMENCLATURE

ρ--gas density (kg/m^3) μ—
absolute viscosity (Pas)
u—x-component of flow velocity (m/s)
v—r-component of flow velocity (m/s)
$\bar{u} - average\ \ flow\ velocity\ \ (m/s)$
u_{max}—maximum flow velocity (m/s)
Q—flow throughput (m^3/s)
V==flow velocity (m/s)
ΔP—line pressure drop (N/m^2)
L—line length (m) D—
nominal pipe diameter (m) R—
pipe radius (m)
r—pipe radial positioning from the center line of the pipe (m)
x—pipe axial positioning along the center line (m)
h—specific enthalpy of the gas
$\bar{h}$ --Absolute enthalpy of the gas

INTRODUCTION

Gas pipeline pressure-flow problem could adequately be tackled by computational approach applying the well known gas equations such as Weymouth, panhandle A and Modified Panhandle B equations [Shadrack, M. U & Abam, D. P. S (2013), Abam, D. P. S. & Shadrack M. U. (2013)]. Theoretical equations governing pressure-flow-temperature relationships revolves often around the Navier Stokes equations when reduced to their appropriate forms with applicable assumptions. The gas pipeline in view is modeled as horizontal system of constant nominal diameter, D. The upstream and downstream pressure and are P1 and P2 respectively. The flow situation is steady state with continuous flow of fluid stream through the pipeline. Fig. 1 is the geometric representation of the system under consideration.

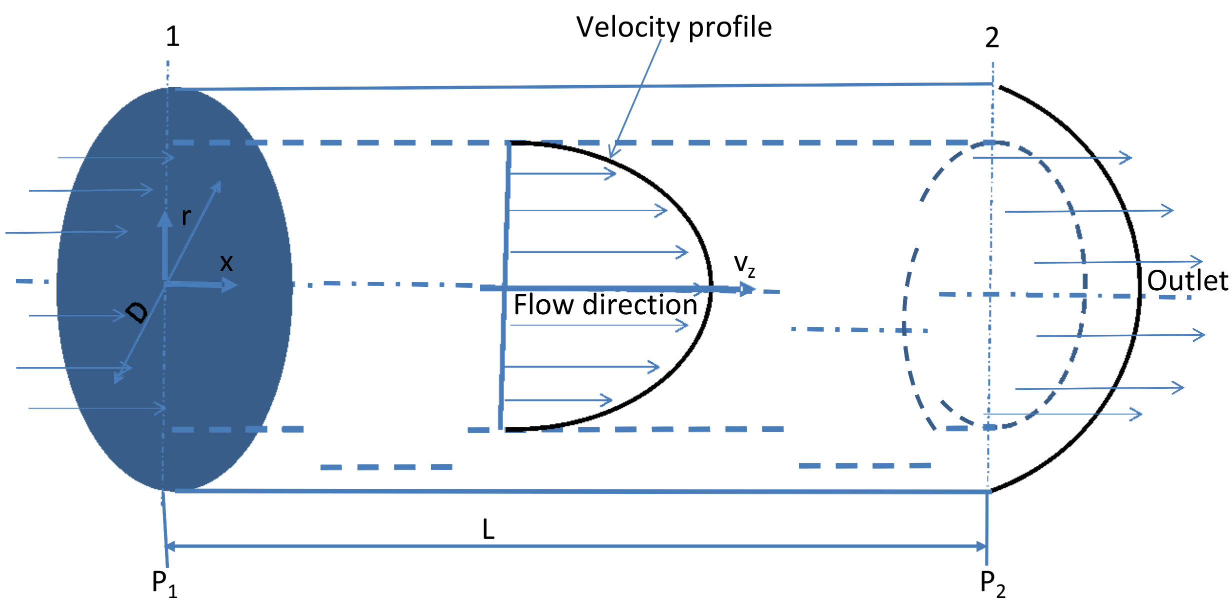


Fig. 1 : Schematic Configuration of The Pipeline

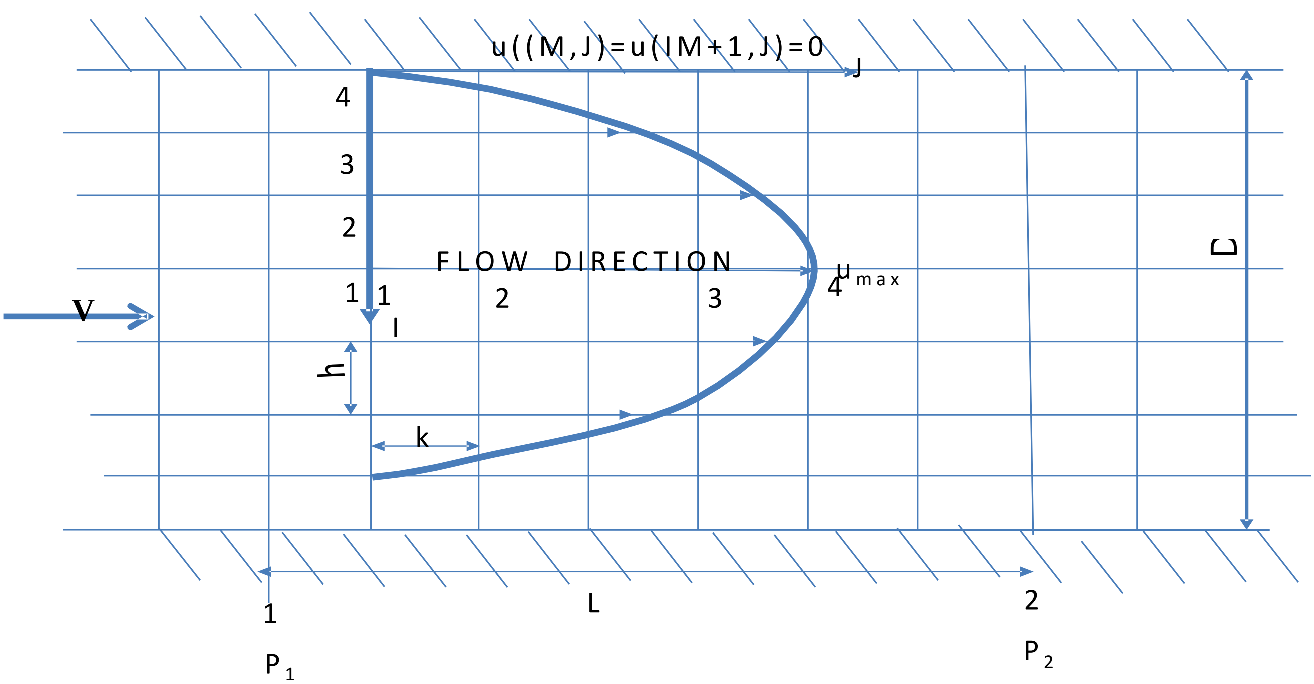


Fig. 2: Physical representation of The Flow Pattern

Deduction for flow throughput and velocity are geared toward formulating more simplified models that could be handled analytically or by numerical discretization approach. The throughput and/or the flow velocity so obtained by this analysis could be a turn around point to comparing the operational conditions and the theoretical situations in a bid to making or operating gas pipeline network system perform optimally.

RESEARCH SIGNIFICANCE

Over the years experience and research revealed the operating gas pipelines in Nigeria terrains operate significantly below optimal level of performance. This trend is absolutely not encouraging imagining the huge capital investment on large tonnage iron and steel. Exploring theoretical background of flow conditions in gas pipelines would create better understanding of flow processes subject to the real flow situations under operating conditions. It is believed that this study could go along way to upgrading the design and the operation parameters of our existing pipelines to make them function optimally.

MODEL DEDUCTIONS

The Navier Stokes equations for mass, momentum and energy conservation applied to steady, axis-symmetric flow in pipelines and expressed in cylindrical coordinates are in the form:

Conservation of mass

$$\frac{\partial}{\partial x}(\rho u r)+\frac{\partial}{\partial r}(\rho v r)=0 \qquad (1)$$

Conservation of momentum perpendicular to principal flow direction

$$\rho v r\frac{\partial v}{\partial r}+\rho u r\frac{\partial v}{\partial x}=r\frac{\partial}{\partial r}(rP)+\frac{\partial}{\partial x}\left(r\mu\frac{\partial v}{\partial x}\right)+\frac{1}{r}\frac{\partial}{\partial r}\left(r\mu\frac{\partial v}{\partial r}\right)+\frac{\partial}{\partial r}\left(r\mu\frac{\partial v}{\partial r}\right). \qquad (2)$$

Conservation of momentum along the principal flow direction

$$\rho v r\frac{\partial u}{\partial r}+\rho u r\frac{\partial u}{\partial x}=-r\frac{\partial}{\partial x}(rP)+\frac{1}{r}\frac{\partial}{\partial r}\left(r\mu\frac{\partial u}{\partial r}\right)+\frac{\partial}{\partial x}\left(r\mu\frac{\partial u}{\partial x}\right)-\rho g r \qquad (3)$$

Conservation of energy

$$\frac{\partial}{\partial x}(r\rho u h)+\frac{\partial}{\partial x}\left(r\lambda\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial r}(r\rho v h)+\frac{\partial}{\partial r}\left(r\lambda\frac{\partial T}{\partial r}\right)+\frac{\partial}{\partial r}\left(r\mu\frac{\partial u^2/2}{\partial r}\right)+\frac{\partial}{\partial x}\left(r\mu\frac{\partial u^2/2}{\partial x}\right)+\frac{\partial}{\partial r}\left(r\mu\frac{\partial v^2/2}{\partial r}\right)+\frac{\partial}{\partial x}\left(r\mu\frac{\partial v^2/2}{\partial x}\right)=0 \qquad (4)$$

Where $h=h+\frac{1}{2}u^2$

h—specific enthalpy of the gas

$\bar{h}$ --Absolute enthalpy of the gas

Applying Navier Stokes equation as gas pipeline solution method to determine flow velocity and flow throughput, the following simplifying assumptions are required. The geometric configuration of the system in view and pattern of flow are as represented in Fig. 1 and Fig. 2 respectively.

(i) The flow is axis-symmetric, i.e., the principal flow direction is x-direction.
(ii) There is no gradient of velocity in r-direction, i.e., $\partial v/\partial r=0$. The implication is that across any section of the pipeline perpendicular to the axis of the pipeline, flow velocity must remain a constant.
(iii) The pressure force component, ρgh, is negligible compared to the applied gradient of pressure, $\partial P/\partial x$, along the length of the pipe. Therefore, $\rho g=0$.
(iv) Across any section of the pipe, pressure is assumed constant, implying that there is no pressure variation in the radial direction.
(v) The pipe is of uniform cross sectional area or the nominal diameter, D, is constant. Hence, the radius r of the pipe does not depend on the distance, x, along the axis of the pipe.
(vi) Since there are no reactive components or species in the mixture and the flow is axis-symmetric, the Prantdl number, Pr=1.
(vii) The flow energy contribution by virtue of flow velocity, $U^2/2$, is negligible compared to the specific enthalpy or total enthalpy of the fluid stream, h.
In the same vein, $\frac{\partial}{\partial r}\left(r\mu\frac{\partial}{\partial r}\left(\frac{u^2}{2}\right)\right)$, compared to $\frac{\partial}{\partial r}\left(r\mu\frac{\partial h}{\partial r}\right)$ is negligible.
(viii) The radial velocity component, v, is not significant compared to the axial velocity component, u.
(ix) It is assumed that the density of the fluid stream is constant and the gas pipeline pressure flow is fully developed turbulent flow at steady state.
(x) The average velocity of the fluid stream is constant.
(xi) The velocity of the fluid stream at boundary of the annular pipe is zero, i.e., u=0 at r=R=D/2.
(xii) The maximum value of flow velocity subject to the physical configuration of the pipeline is finite and occurs along the center line of the pipes. For this situation to hold, $\partial u/\partial r=0$ at r=0. Hence, the velocity profile is parabolic in nature.

Subject to these assumptions, the equations reduce to the form :
(i) The differential form of the continuity equation for compressible flow is expressed as :

$$\frac{\partial}{\partial x}(\rho ur)+\frac{\partial}{\partial r}(\rho vr)=0. \tag{5}$$

(ii) The differential form of the momentum equation is expressed as:

$$\rho u\frac{\partial u}{\partial x}+\rho v\frac{\partial u}{\partial r}=-\frac{\partial P}{\partial x}+\frac{1}{r}\frac{\partial}{\partial r}\left(r\mu\frac{\partial u}{\partial r}\right)-\rho g \qquad (6)$$

Applying the conditions in (ii) and (iii),

$$\rho u\frac{\partial u}{\partial x}+\frac{1}{r}\frac{\partial}{\partial r}\left(r\mu\frac{\partial u}{\partial r}\right)=\frac{\partial P}{\partial x} \qquad (7)$$

(iii) The differential form of the energy equation goes thus,

$$\rho u\frac{\partial h}{\partial x}+\rho v\frac{\partial h}{\partial r}=\frac{1}{r}\frac{\partial}{\partial r}\left(r\mu\frac{\partial h}{\partial r}\right) \qquad (8)$$

Applying also the conditions in (ii) and (iii)

$$\rho u\frac{\partial h}{\partial x}=\frac{1}{r}\frac{\partial}{\partial r}\left(r\mu\frac{\partial h}{\partial r}\right). \qquad (9)$$

Adding equations (7) and (9),

$$\rho u\frac{\partial}{\partial x}(u+\alpha h)-\frac{1}{r}\frac{\partial}{\partial r}\left(r\mu\frac{\partial}{\partial r}(u+\alpha h)\right)=\frac{\partial p}{\partial x} \qquad (10)$$

By analytical or numerical approach these equations can be manipulated to determine the flow velocity expressed in the form;

$$u=u\left(x,\ h,\ r,\ \mu,\ \frac{\partial P}{\partial x}\right). \qquad (11)$$

Subsequently, the flow throughput, Q, could be obtained by taking the product of the flow velocity, u, and the pipe area of cross section, A.

Q=uA (12)

To determine the flow velocity, the flow situation is assumed being at steady state, incompressible and fully developed. The continuity equation 1 reduces to the form :

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial r} \qquad (13)$$

For axis-symmetric flow , velocity component in the radial direction, v, is negligible, thus, v=0 and $\partial v/\partial r=0$, therefore,

$$\partial u = 0 \tag{14}$$

Applying the condition in equation (viii) to the momentum equation in 3,

$$\rho u \frac{\partial u}{\partial x} + \frac{1}{r}\frac{\partial}{\partial r}\left(r\mu \frac{\partial u}{\partial r} \right) = \frac{\partial P}{\partial x} \tag{15}$$

$$\frac{\partial}{\partial r}\left(r \frac{\partial u}{\partial r} \right) = \frac{1}{\mu}\left(\frac{\partial P}{\partial x} \right) r \tag{16}$$

Integrating equation 16 with respect to r,

$$r\frac{\partial u}{\partial r}=\frac{1}{\mu}\left(\frac{\partial P}{\partial x}\right)\frac{r^2}{2}+c_1$$

$$\frac{\partial u}{\partial r}=\frac{1}{\mu}\left(\frac{\partial P}{\partial x}\right)\frac{r}{2}+\frac{c_1}{r}$$

Integrating once more with respect to r,

$$r\frac{\partial u}{\partial r}=\frac{1}{\mu}\left(\frac{\partial P}{\partial x}\right)\frac{r^2}{2}+c_1$$

$$u=\frac{1}{\mu}\left(\frac{\partial P}{\partial x}\right)\frac{r^2}{4}+c_1\ln r+c_2$$

To evaluate constants c_1 and c_2, the boundary conditions stipulate that u=0 at r=R=D/2; hence there is only one boundary condition. The velocity at the pipe center line (r=0) is unknown, but by physical considerations this velocity should be finite at r=0. The only feasible approach then is that $c_1=0$, thus:

$$u=\frac{1}{\mu}\left(\frac{\partial P}{\partial x}\right)\frac{r^2}{4}+c_2 \qquad 1$$

(7)

Applying the boundary condition,

$$c_2=-\frac{R^2}{4\mu}\left(\frac{\partial P}{\partial x}\right)$$

Substituting for c_2 in equation 17

$$u=\frac{1}{4\mu}\left(\frac{\partial P}{\partial x}\right)\left(r^2-R^2\right) \qquad (18)$$

For fully developed flow equation 18 gives the profile of the velocity distribution, which is parabolic in nature as earlier predicted.

The volumetric flow rate of the fluid is expressed as:

$$Q+\int_0^R udA=\int_0^R u\times 2\pi r dr \qquad (19)$$

Substituting equation 18 in 19

$$Q=\int_0^R\frac{1}{4\mu}\left(\frac{\partial P}{\partial x}\right)\left(r^2-R^2\right)2\pi r dr$$

$$=-\frac{\pi R^4}{8\mu}\left(\frac{\partial P}{\partial x}\right) \qquad (20)$$

The flow rate as a function of pressure drop could be obtained by considering a fully developed turbulent flow, whereby the pressure gradient, ∂P/∂x, is constant. In this light,

$$\frac{\partial P}{\partial x}=-\frac{P_1-P_2}{L}=-\frac{\Delta P}{L}. \qquad (21)$$

Substituting equation 21 in 20

$$Q = -\frac{\pi R^4}{8\mu}\left(\frac{\partial P}{\partial x}\right)$$
$$= -\frac{\pi R^4}{8\mu}\left(-\frac{\Delta P}{L}\right)$$
$$= \frac{\pi D^4}{128\mu}\left(\frac{\Delta P}{L}\right) \qquad (22)$$

Hence,for laminar flows in pipes, the average flow velocity is expressed:

$$\bar{u} = \frac{Q}{A} = \frac{R^2}{8\mu}\left(\frac{\Delta P}{L}\right) \ldots\ldots (18)$$

To determine the maximum flow velocity and the radial position, ∂u/∂r is set to zero.

$$u = \frac{1}{4\mu}\left(\frac{\partial P}{\partial x}\right)\left(r^2 - R^2\right) \qquad (18)$$

$$\frac{\partial u}{\partial r} = \frac{1}{4\mu}\left(\frac{\partial P}{\partial x}\right)2r = 0$$

$$\therefore \quad r = 0$$

$$u_{max} = \frac{R^2}{4\mu}\left(\frac{\Delta P}{L}\right) \qquad (23)$$

$$u_{max} = 2\bar{u}$$

Point of maximum velocity occurs at the center line of the pipe and the maximum value is twice the average flow velocity.

RECOMMENDATION

Gas pipeline pressure-flow subject to flow velocity and throughput should be addressed by computational approach. This is with the view of forming a sound theoretical and practical understanding of flow in gas pipelines.

CONCLUSION

Navier Stokes equations reduced to their appropriate forms by practicable simplifying conditions had been deduced. This is to enable better understanding of axis-symmetric flow in gas pipelines subject to theoretical and practical flow situations.

REFERENCES

SHADRACK, M. U.& ABAM, D. P. S. (2013). Flow Optimization Models in Gas Pipelines (Modified Panhandle-B Equation As Base Equation), Journal of Science and Technology Research. Vol. 6, No. 1, Pp 31-41, April 2013.

SHADRACK, M. U. & ABAM, D. P. S. (2013). Flow Optimization Models in Gas Pipelines (Weymouth Equation As Base Equation), African Science and Technology Journal Siren Research Centre for African Universities. Vol. 6, No. 1, Pp 109-123, April 2013.

ABAM, D. P. S. & SHADRACK, M. U. (2013). Flow Optimization Models in Gas Pipelines (Panhandle-A Equation As Base Equation), Journal of Science and Technology Research. Vol. 6, No. 3, Pp 1-16, December, 2013.