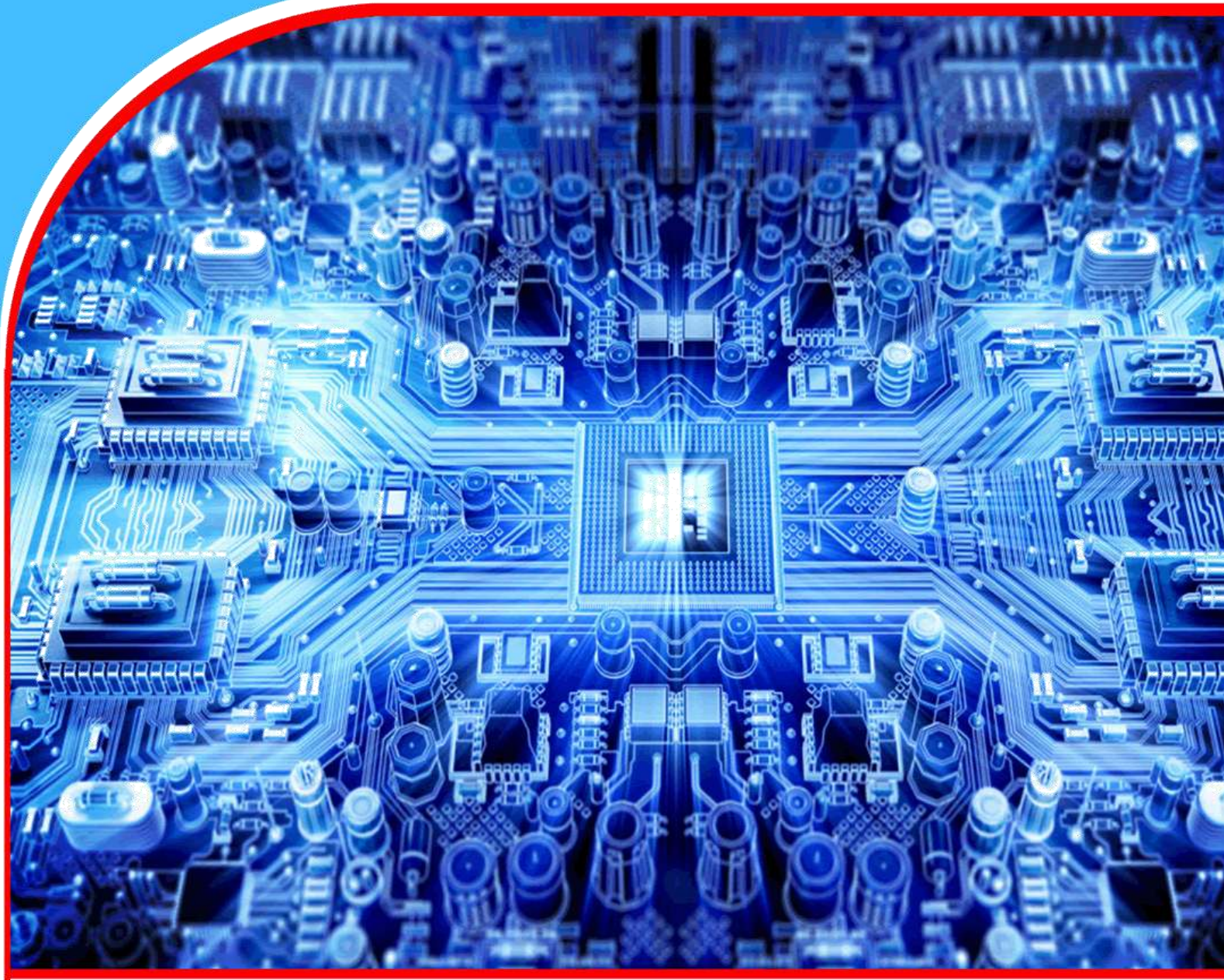
American Journal of
Computing and Engineering
(AJCE)



MATHEMATICALLY MODELING THE SPREAD OF HIV/AIDS INFECTION AFTER THE INTRODUCTION OF ANTIRETROVIRAL THERAPY IN GHANA

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American Journal of Computing and Engineering
ISSN 2520-0449 (Online)
Vol.3, Issue 1, pp 24-39, 2020
www.ajpojournals.org

MATHEMATICALLY MODELING THE SPREAD OF HIV/AIDS INFECTION AFTER THE INTRODUCTION OF ANTIRETROVIRAL THERAPY IN GHANA.

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ABSTRACT

One of the deadliest and highly infectious diseases is Human Immunodeficiency Virus-Acquired Immune Deficiency Syndrome (HIV-AIDS). One person was first diagnosed with HIV-AIDS in the Eastern region part of Ghana in 1986, followed by 41 more in the same year. The spread of the disease was so unbearable in the subsequent years (1987-2003), even after several intervention measures taken by Government and other stakeholders. Hence, in 2003, the antiretroviral therapy (ART) program was introduced by the Ghana AIDS Commission (GAC) through the Ghana Health Service (GHS) to sabotage the virus such that it cannot be transmitted from one person to another, not even from the pregnant mother to the unborn child. It is in this direction that this research was conducted to use differential equations to derive a model for the prediction of the HIV/AIDS infection rate, after the introduction of the ART program in Ghana. The data on the number of HIV infected people per each year (I) for the years 2003-2018 were collated from the reports given by Ghana AIDS Commission (GAC), WHO and UNAIDS published on their associate websites. Differential equations, with the employment of numerical analysis of data, were used to derive a model for the prediction of the yearly number of HIV/AIDS infected people. Graphical analysis on the residuals of the predicted number of HIV/AIDS infected people (residual analysis) were carried out to check whether the derived model was adequate or not. Finally, a model was derived using ordinary differential equations and the yearly numbers of HIV infected people estimated using the model were in descending, order as portrayed in the original data set. The residual analysis on the model adequacy checking proved that the model is adequate for the prediction of the number of HIV infected people in Ghana. In effect, the ART program really played a major role in the reduction of the HIV infection rate. The uniqueness of this research is portrayed in the fact that it is the first time differential equation is being employed in Ghanaian academia to derive a model for the future prediction of the HIV infection rate.

Keywords: *Human Immunodeficiency Virus, Acquired Immune Deficiency Syndrome, antiretroviral therapy, HIV-Infected People, Susceptible People.*

1. INTRODUCTION

1.1 **What is Hiv/Aids?**

Human Immunodeficiency Virus (HIV) is the virus that causes Acquired Immune Deficiency Syndrome (AIDS). The virus acts by weakening the immune system, making the body susceptible to other diseases and unable to recover from other such diseases as pneumonia, tuberculosis (TB), etc. A person can be infected with HIV for a long time without showing any symptom of the disease. Nonetheless, during that period before a person develops symptoms, he/she can transmit the infection through sex, child delivery, breastfeeding, blood transfusions, sharing of injection needles, etc. However, an individual is said to have developed AIDS when he/she presents with a combination of signs and symptoms after being tested HIV positive. This is as accounted by National AIDS/STI Control Program of the Disease Control Unit.

(National AIDS/STI Control Program, 2001)

1.2 **Brief History of Hiv/Aids in Ghana**

In March 1986, the first case of HIV/AIDS was reported in the Eastern region of Ghana. In January 1991, a more detailed report on HIV/AIDS in Ghana appeared in which 107 positive cases were recorded in 1987 and this increased to 333 by the end of March 1988. There was a tremendous increment of this to 2,744 by the end of April 1990, out of which 1,226 were said to have migrated to AIDS status. From the World Health Organization (WHO) annual report, the spread of HIV/AIDS was tremendous to the extent that Komfo Anokye Teaching Hospital alone was recording an average 50 cases monthly in 1991. Not all, following the introduction of an improved reporting mechanism, Ghana recorded 12500 AIDS cases by the end of 1994, placing Ghana second to Côte d'Ivoire in the West Africa sub region, where more than 16600 cases were recorded. (The Library of Congress Country Studies, 2020).

1.3 **Interventions**

Right from the beginning, the various governments and other stakeholders have undertaken some intervention measures to help cap the HIV/AIDS situation in Ghana. In 1985 for instance, an advance step was taken by the government through the formation the Ghanaian AIDS Advisory Committee to council and direct the government on the prevention of HIV transmission. Another unit, the 1989 Medium Term Plan, was developed to support the creation of HIV surveillance systems: information, education and communication (IEC) efforts; testing of blood supply; psychosocial support program for HIV-infected people; and adequate clinical management of HIV/AIDS cases. This was followed by the formation of the National Advisory Council on AIDS (NACA) to guide in the establishment of AIDS policy development. (Amofa, 1992).

However, the HIV/AIDS continued to be generalized as epidemic because the prevalence rate was more than 1%. The government then responded by forming the Ghana AIDS Commission (GAC) in 2002 by Public Act 2016, Act 938 of Parliament whose function, together with other

stakeholders, was to help deal with the situation. It was through the effort of GAC, with the support of the government, Ghana Health Service (GHS) and WHO that the anti-retroviral therapy (ART) was introduced in 2003. (Aboagye-Sarfo, 2015).

The ART is a system of administering drugs to HIV-infected patients to attack the HIV at different stages of its life cycle to inhibit the virus replication, and thus bring the viral load down. According to WHO, standard anti-retroviral therapy (ART) consists of the combination of antiretroviral (ARV) drugs to maximally suppress the HIV virus and stop the progression of HIV disease. (WHO, 2020).

Various researches have been conducted elsewhere to address some issues concerning ART in relation to HIV/AIDS infections and other related matters, with most of them focusing on derivation of mathematical models. Jinghua, Liping, Stuart,et al. (2018) for instance used Ordinary Differential Equations to develop a deterministic mathematical model and projected over 20 years to assess the impact of the Pre-exposure prophylaxis (PrEP), test-and-treat and their combinations on the cost-effect and the risk of HIV transmission among men who have sex with men (MSM) in China, and they found that these methods were effective and cost-effective relative to current policy in China. Also, Li and Wang (2006), in their study, performed a nonlinear analysis on backward bifurcation in a mathematical model for HIV infection with ART treatment through which they discovered a parameter regimen for which backward bifurcation can occur to explain the sudden rebound of HIV viral load when ART is stopped, and also provide an explanation for the viral blips during the ART suppression on HIV. Omondi, Mboyo and Luboodi (2018) used Lyapunov function to derive mathematical models to determine the quantification of HIV prevention, testing and treatment with ART in Kenya. They also carried out a sensitivity analysis to determine the model parameters for HIV transmission, through which it was detected that the effective contact rates are the mechanisms fuelling HIV epidemic proliferation.

In their study, Oladotun and Suarez (2020) employed the Gauss-Seidel-like implicit finite-difference (GSS1) method to solve their proposed mathematical model containing two optimal control variables: the HIV uninfected $CD4^+$T cells and the HIV infected $CD4^+$T cells. They then performed logistic growth analysis on the proposed model (validated by numerical simulation) and the results showed that concentration of the HIV uninfected $CD4^+$T cells was maximized while that of the HIV infected $CD4^+$T cells was minimized in the body using minimum drug therapies. Rivadeneira, et al (2014) also used optimal control theory and differential equation to model the side effects produced by HAART and found out that the positive side supersedes the negative side at a minimum monetary cost.

The purpose of this research is that, with the introduction of the ART program in 2003, this research is directed towards the derivation of mathematical model, using ordinary differential equations, for the prediction of HIV infection rate, hence the assessment of the ART program's effectiveness.

2. METHODOLOGY

2.1 **Data Collection**

The data for the study was collected from google scholar websites. For instance, the estimated Ghanaian population recorded in the corresponding years (2003 to 2018) was retrieved from Worldometer (2019). The data on the number of HIV infected people per each year (*I*) for the years 2003-2018 were collated from the reports given by Ghana AIDS Commission (GAC), WHO and UNAIDS published on their associate websites. The national HIV prevalence rate per the corresponding years is based on the HIV Sentinel Surveys (HSS) report on the HIV prevalence among pregnant women antenatal clinic (ANC) attendants at 40 sentinel sites across the regions of Ghana, as approved by the Ghana Health Service (GHS) and GAC (National AIDS/STI Control Program, 2019). The yearly number of susceptible people (*S*) was also determined by finding the difference between the number of HIV infected people (*I*) and the estimated population of the year, that is *S* = Population – *I*. The number of HIV infected people who have been enrolled onto the ART program and as such removed (*R*) from the susceptible group was also retrieved from GAC annual reports.

2.2 **Model Formulation**

One particular type of model is differential equation which is an equation that contains an unknown function and some of its derivatives, by Stewart (2006). In a real-world, it is the application of calculus in formulating a mathematical model to predict future behavior of a phenomenon on the basis of how current values change. According to Backhouse, Houldsworth, and Cooper (1985), a differential equation is an equation containing such differential coefficients as $\frac{dy}{dx}, \frac{d^2y}{dx^2}, \frac{d^3y}{dx^3}$, etc.

Differential equations were used to derive the mathematical model for the prediction of yearly HIV-infected people after the introduction of the antiretroviral therapy (ART). To effectively apply differential equations in the model formulation of the ART program effect on the HIV infection rate, the entire Ghanaian population is divided into four groups:

S= the number of ***susceptible*** people, that is people who are not yet infected with HIV but could be infected.

I = the number of people who are ***infected*** or living with HIV but are **not** on the ART program.

Q= the number of people who are ***infected*** or living with HIV, including those on the ART program.

R= the number of people who are living or infected with HIV and are ***removed*** from the susceptible group (*S*) as a result of being enrolled onto the ART program.

NB: once a person gets onto the ART and continue to stay on it, he or she cannot infect others with HIV any longer.

The number of susceptible people (*S*) changes in two ways:

a) The yearly population growth adds on to S.
b) The newly HIV infected people leave S.

In the case of situation ***(a)***, the new people that cause the population to grow are the same people that add onto the number of susceptible people (*S*), hence accrue the rate *xS*, where *x*>0 is the yearly population growth rate.

In the case of situation ***(b)***, let's assume the rate susceptible people become infected with HIV is proportional to the number of contacts (by blood or body fluid) between susceptible people and HIV infected people. Hence, it is expected that the number of contacts (by blood or body fluid) between the two groups is proportional to both *S* and *I*. This implies that the number of people leaving *S* as a result of HIV infection accrues the rate *ySI*, where *y* is the rate of HIV infection yearly.

Combining situations ***(a)*** and ***(b)***, the rate of changes in the number of susceptible people is equal to the rate *S* grows minus the rate HIV infected people leave *S*.

$$=>\frac{dS}{dt} = xS - ySI \ldots\ldots\ldots\ldots (1)$$

The number of people who are living with HIV, including those on the ART program (*Q*) also changes in two ways:

c) The newly HIV infected people add onto Q.
d) The people who die out of HIV-AIDS leave Q

With situation ***(c)***, the number of the HIV infected people, including those on ART program (*Q*) is also proportional to the number of contacts (by blood or body fluid) between susceptible people and HIV infected people who are not on ART program (*I*). This also accrues at the rate *ySI*.

With situation ***(d)***, assuming those on ART and die of HIV-AIDS is negligible, as such only those who are not on ART are considered, the number who die of HIV-AIDS yearly accrues at the rate *zI*, where z>0 is the HIV-AIDS death rate.

The rate of change in Q= $ySI-zI$

$$=>\frac{dQ}{dt} = ySI - zI \ldots\ldots\ldots\ldots (2)$$

Also, the number of HIV infected people who are not on ART program (I) changes in two ways:

e) *The newly HIV infected people add onto I.*
f) *Others are **removed** from I as a result of being on the ART or dead out of HIV-AIDS.*

Since the number of the newly HIV infected people who add onto I is the same as those who add onto Q, situations (e) and (c) are equal. This implies that (e) also accrues at the rate ySI.

With situation ***(f)***, the more people get infected with HIV, the more people get onto the ART program or die of HIV-AIDS (especially those who are not treated with ART). Hence, people are removed from I(R) at a rate proportional to the number of HIV infected people who are not on ART (I). This also accrue at a rate mI, where m>0 is the rate people are removed from I.

$$=>\frac{dR}{dt} = -(\text{Rate HIV infected people are remved from } I) = -mI \ldots\ldots\ldots\ldots (3)$$

(The negative sign is used because I is decreasing)

Now combining situations (e) and (f), the rate of change in I = (Rate people leave S) − (Rate HIV infected people are removed from I)

$$=>\frac{dI}{dt} = ySI - mI \ldots\ldots\ldots\ldots\ldots\ldots (4)$$

Before concluding on the accuracy of the derived model, the adequacy of the model should be checked. The primary diagnostic tool for model adequacy checking is the residual analyses which were mostly done by graphical analysis in different forms and simply called residual plots. In Walpole et al., (2007), a residual is essentially an error in the fit of a model.

The residuals for t model are given by:

$$E = I - \hat{I},$$

where $\hat{I}$ is the estimator of I.

For the residual analysis, the normal probability plot was used for the model adequacy checking. In this case, the residuals are scaled on the horizontal axis as against the HIV prevalence rate (%) on the vertical axis for the years 2003 to 2018, using Excel software installed on my personal computer. For the normal probability plot of residuals, if the underlying error of the distribution is normal, then the plot exhibits some kind of linearity.

American Journal of Computing and Engineering
ISSN 2520-0449 (Online)
Vol.3, Issue 1, pp 24-39, 2020
www.ajpojournals.org

According to Montgomery (2001), if the model is adequate, the residuals should be structureless; that is, they should contain no obvious patterns. However, a very common defect that often shows up on the normal probability plots is one residual being much larger than the others, and this can seriously distort the conclusion drawn on the model adequacy. This residual is called an outlier. Mostly, the cause of the outlier is such human error as calculation error, date coding error or copying error. However, a suspected outlier could be checked by examining the standardized residuals value (d), given by:

$$\text{Standard residual value } (d) = \frac{E}{\sqrt{\sum E^2}}$$

A standard residual value (d) bigger than 3 in absolute is a potential outlier which can cause a serious distortion to the conclusion drawn on the model adequacy.

3. RESULTS

Table 1 below generally illustrates data set of the number of HIV infected people (I) and those removed from the infected people recorded for the period 2003-2018. The Ghanaian population (P) per year for the period 2003-2018, as retrieved from Worldometer, is as shown the third column. It is then followed by the yearly number of HIV infected people (I) recorded in the respective years. The fifth column is headed by the number of susceptible people calculated by using the relation $S = P - I$. The sixth column is headed by the number of HIV infected people who have been enrolled onto the ART program and as such removed (R) from the susceptible group. The last column shows the yearly HIV prevalence rate, that is how quick people are infected with HIV every year, as reported by HSS.

Table 1: the number of HIV infected people (I) and those removed from the infected people recorded for the period 2003-2018

Year	Time (in years)	Population (P)	I	$S = P - I$	R	HIV Prevalence Rate (%)
2003	1	20301686	736839	19564847	197	3.6
2004	2	20835514	645901	20189613	2028	3.1
2005	3	21884034	577369	20806665	4060	2.7
2006	4	21947779	702328	21245451	7338	3.2
2007	5	22525659	585667	21939992	13429	2.6
2008	6	23110139	241538	22868601	23614	2.2
2009	7	23691533	267069	23424464	33745	2.9
2010	8	24262901	221941	24040960	45226	2.0

2011	9	24791070	261925	24529145	62862	2.1
2012	10	25366462	235982	25130480	73339	2.1
2013	11	25904598	224488	25680110	67346	1.9
2014	12	26107155	227406	25879749	75044	1.6
2015	13	27849205	274078	27597308	69616	1.8
2016	14	28481945	294720	28228644	83111	1.8
2017	15	29121465	314782	28866478	97582	2.1
2018	16	29767102	334713	29547171	113133	1.6

3.1 **Estimation of the Parameters *x, y, z* and *m***

Estimating the parameters x, y, z and m in equations (1), (2), (3) and four above, the following procedures are involved.

For *x*: *x* measures how quick the Ghanaian population grows annually. Based on worldometer elaboration of United Nations data, the annual population growth rates of Ghana from 2003 to 2018 are 1.45%, 1.36%, 1.25%, 2.07%, 1.97%, 1.93%, 1.88%, 1.86%, 1.82%, 1.79%, 2.19%, 2.19%, 2.36%, 2.27%, 2.25% and 2.22% respectively. Hence, on the average: the population growth rate is:

$$x \approx 1.93\% \approx \mathbf{0.0193}$$

For *y*: *y* measures how infectious HIV is, that is how quickly HIV is transmitted from the infected people to the susceptible people. From table 1, the Ghanaian population in 2003 and 2018 were 20301686 and 29767102 respectively. Also, about 736839 and 219931 people were living with HIV in the years 2003 and 2018 respectively. Not all, the number of susceptible people (*S*)=**19564847** in 2003 (year 1) and (*S*)=**29547171** in 2018 (year 16).

$$=>\frac{dS}{dt} = \frac{29547171-19564847}{16-1} \approx \mathbf{665488}$$

Also, in the year 2003 that the ART program was initiated, 197 people got onto the ART program (GAC, 2013). Hence, $I = 736839 - 197 = \mathbf{736642}$.

Now putting $\frac{dS}{dt} = \mathbf{665488}$, $x \approx \mathbf{0.0193}$, $S = \mathbf{19564847}$ and $I = \mathbf{736642}$ into equation (1) above,

$$=> 665488 = (0.0193)(19564847) - y(19564847)(736642)$$

$$=>y = \frac{(0.0193)(19564847) - 665488}{(19564847)(736642)} \approx \mathbf{-0.00000002.}$$

For *z*: *z* measures how fast people die of HIV-AIDS yearly. From GAC's accounts, within the period 2003-2018, the highest number of people who lived with HIV (*Q*) was *Q* = **736839** in 2003(year 1) and the lowest number recorded was *Q* = **219931** in 2018 (year 16).

$$=>\frac{dQ}{dt} = \frac{736839-219931}{1-16} \approx \mathbf{-34461}$$

Substituting $\frac{dQ}{dt} = -36922$, *S*= 19564847, *I* = 736642 (recorded in 2003) and *y* = −0.00000002 into equation (2),

$=>-34461 = (-0.00000002)(19564847)(736642) - Z(736642)$

$$=>Z = \frac{34461 - (0.00000002)(19564847)(736642)}{736642} = \mathbf{-0.3445}.$$

For m: *m* measures how quick HIV infected people are removed from *I* as a result of being enrolled onto the ART program or death. With reference to the values of *Q* provided in 2003 and 2018, the corresponding number of HIV infected people who were enrolled onto the ART (i.e. *R*) were 197 and 61188. This gives the *I* values as *I* = 736839−197 = **736642** in 2003 (year 1) and *I* = 334713−113133 = **221580** in 2018 (year 16).

$$\because \frac{dI}{dt} = \frac{736642-221580}{1-16} \approx \mathbf{-34337}$$

Putting $\frac{dI}{dt} = -34337$, *S*= 19564847, *I* = 736642 (recorded in 2003) and *y* =−0.00000002 into equation (4),

$=>-34337= (-0.00000002)(19564847)(736642)-m(736642)$

$=>m = \frac{34337 - (0.00000002)(19564847)(736642)}{736642} = -\mathbf{0.3447}.$

Substituting the values of the constants *x*, *y*, *z* and *m* into their respective positions in equations (1), (2), (3) and (4), the differential equations obtained are:

$$\frac{dS}{dt} = 0.0193S + 0.00000002SI \text{.....(5)}$$

$$\frac{dQ}{dt} = -0.00000002SI + 0.3445I \text{ .....(6)}$$

$$\frac{dR}{dt} = 0.3447I \text{ .....(7)}$$

$$\frac{dI}{dt} = -0.00000002SI + 0.3447I \text{ .....(8)}$$

Since the study is basically purposed at evaluating the effect of antiretroviral therapy (ART) on the HIV infections in Ghana, let's focus on equations (7) and (8). Relating *I*, *R* and *S* directly, let *I* be a function of *R* and *S*, and *R* and *S* be functions of time (*t*) in years.

By the chain rule:

$$\frac{dI}{dt} = \frac{dI}{dR} \times \frac{dR}{dt}$$

$$=> \quad \frac{dI}{dR} = \frac{dI/dt}{dR/dt}$$

Substituting for dI/dt and dR/dt as in equations (7) and (6)

$$=>\frac{dI}{dR} = \frac{-0.00000002SI + 0.3447I}{0.3447I}$$

$$=>\frac{dI}{dR} = \frac{-S}{17235000} + 1 \text{.....(9)}$$

Equation (9) signifies the change in the number of HIV infected people with respect to the change in the number of HIV infected people being enrolled onto the ART program.

Continuing from equation (9),

$$dI = \left(\frac{-S}{17235000} + 1\right) dR$$

Integrating both sides, i.e.

$$\int dI = \int \left(\frac{-S}{17235000} + 1\right) dR$$

$$=> I = \frac{-SR}{17235000} + R + C \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (10)$$

Where C is a constant.

Putting in the initial conditions I_o = 736642, S_o= 19564847 and R_o = 197.

$$=> 736642 = \frac{-(19564847)(197)}{17235000} + 197 + C$$

$$=> C = 736642 + \frac{(19564847)(197)}{17235000} - 197 = \mathbf{736668.6307}$$

Putting the C value back into (10),

$$=> I = \frac{-SR}{17235000} + R + 736668.6307 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (11)$$

Equation (11) is the final model for determining the effect of the ART program on the HIV infection rate in Ghana. If it happens that the yearly predicted values of I, as calculated using the model, is in descending order, then the effect of the ART program is positive since it is contributing to the reduction in HIV infection rate, otherwise the effect is negative.

3.2 **Model Adequacy Checking**

For the model adequacy checking, the derived model, equation (11), was used to determine the estimated number of HIV-AIDS infected people ($\hat{I}$), approximated to the nearest whole figure. Hence, the residuals ($E = I - \hat{I}$) of the number of HIV-AIDS infected people were also determined. These are as shown in table 2 below:

Table 2: The residuals of the number of yearly HIV-AIDS infected people from 2003 to 2018

Year	Time (in years)	*I*	*S*	R	HIV Prevalence Rate (%)	Estimated I ($\hat{I}$)	Residuals (*E*)
2003	1	736839	19564847	197	3.6	736642	197
2004	2	645901	20189613	2028	3.1	736321	-90420
2005	3	577369	20806665	4060	2.7	735827	-158458
2006	4	702328	21245451	7338	3.2	734961	-32633
2007	5	585667	21939992	13429	2.6	733003	-147336
2008	6	241538	22868601	23614	2.2	728950	-487412
2009	7	267069	23424464	33745	2.9	724550	-457481
2010	8	221941	24040960	45226	2.0	718809	-496868
2011	9	261925	24529145	62862	2.1	710064	-448139
2012	10	235982	25130480	73339	2.1	703071	-467089
2013	11	224488	25680110	67346	1.9	703669	-479181
2014	12	227406	25879749	75044	1.6	699028	-471622
2015	13	274078	27597308	69616	1.8	694813	-420735
2016	14	294720	28228644	83111	1.8	683655	-388935
2017	15	314782	28866478	97582	2.1	670813	-356031
2018	16	334713	29547171	113133	1.6	655850	-421137

From table 2, the estimated number of HIV infected people, as shown in the seventh column, exhibit a downward trend as the years went by. This could signify a positive effect of the ART program on the HIV infection rate.

The Excel output of the normal probability plot of the residuals is as shown in figure 1 below. The vertical axis is scaled with the HIV prevalence rate (%) while the horizontal axis is scaled with the residuals. The scatter plot is the blue dots while the diagonal line is establishing the linearity of the plot.

American Journal of Computing and Engineering
ISSN 2520-0449 (Online)
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www.ajpojournals.org

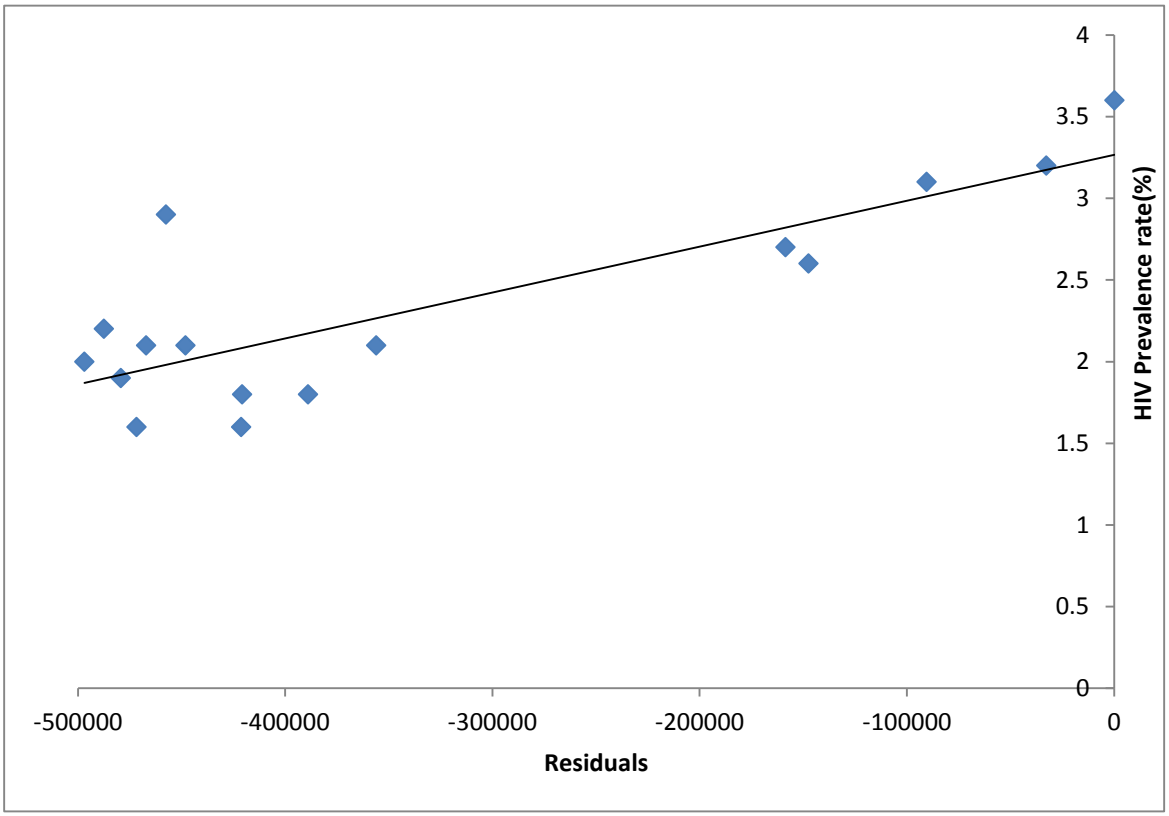

Figure 1: Normal probability plot of residuals of the number of HIV-AIDS infected people

From the graph, figure 1, since the scatter plot is structureless, that is there is no obvious pattern. Not all, visual examination of table 2 and the normal probability plot in figure 1 reveals some extreme residuals, of which the highest is -496868, and this corresponds with the 2010 record (ie E_{2010} = -496868). Finding the standard error (d) of this, we have:

Standard error (d_{2010}) = $\frac{E_{2010}}{\sqrt{\sum E^2}} = \frac{-496868}{\sqrt{(197^2)+\cdots+(-431137)^2}} = \frac{-496868}{1501079.596} \approx$ - 0.331.

4. DISCUSSION

From table 1 above, the population of Ghana kept on increasing year after year and this also caused the number of susceptible people to grow in the same manner. With the introduction of the ART, the number of HIV infected people (I) depreciated within the first three years and then went up drastically in 2006 before exhibiting an up-and-down growth till 2013, where it started trending up gradually. However, with the HIV prevalence rate (%), there was reduction trend for the first three years and then it went up in 2006 after which it exhibited an up-and-down growth for the

subsequent years. The number of people removed from *I* (*R*), as a result of being enrolled onto the ART program, kept on growing up yearly.

Also, in table 2, the estimated number of HIV infected people ($\hat{I}$) is in descending order, which confirms the fact that the ART helps in the gradual reduction of the HIV infection rate, and this is in line with GAC (2015) accession. With these exhibitions, it can happen that, as long as the ART program is highly patronized, there could be a time (*t*) that there would be no new HIV infections, which is in line with Oladotun and Suarez (2020) findings.

For the model adequacy checking, since the scatter plot is structureless, that is there is no obvious pattern, as in figure 1, the model is adequate. The linearity of the plot also proves the normality or independent assumption of the distribution. Not all, since the standard error (d_{2010} = -0.331) calculated is less than 3 in absolute, the error effect is negligible, likewise the other extreme residuals, the model adequacy is confirmed, which follows Montgomery (2001) ideas. However, this error could have been caused by too much approximation during the various levels of calculations and inaccurate recordings at the various sources of the data.

5. CONCLUSIONS

After the introduction of the ART program, the number of the HIV infected people started reducing from 736839 in 2003 in an up-and-down manner in the subsequent years. The HIV prevalence rate also behaved in the same manner as the years went by. However, there was a general downward trend for the HIV infection rate. This is an indication that the introduction of the ART has helped in the reduction of the HIV infection rate in Ghana.

Moreover, the model adequacy checking carried out proved that the derived model:

$$I = \frac{-SR}{17235000} + R + 736668.6307$$

is appropriate and adequate for the prediction of the number of HIV-infected people per year. Hence, can be used for the determination of the effect of ART program on HIV-AIDS infection rate in Ghana.

6. RECOMMENDATIONS

Following the success of this study, it is recommended that the government, through the Ghana AIDS Commission, should provide a much improved mechanism for implementation of the ART program and also organized effective educational programs to enlighten the people, especially those with HIV infection, on the patronage of the ART program. The public too should be educated on the need to avoid stigmatization, as it is one of the reasons why people find it difficult to declare their HIV status for the necessary measures to be taken.

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